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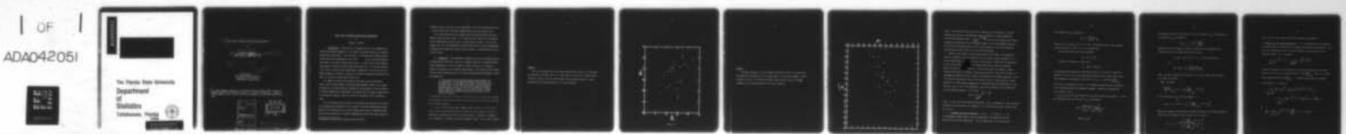

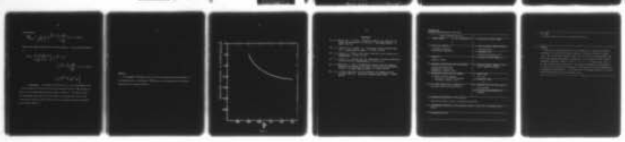

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by

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Morgan A. Hanson

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Rank Tests in Weather Modification Experiments

by

Morgan A. Hanson*

1. Introduction. Rank tests are frequently used in the comparison of seeded and unseeded responses or of control and target responses in weather modification experiments. The difficulty in characterizing rainfall distribution makes such nonparametric tests desirable. One of the more frequently used tests is the sum of squared ranks test devised by Taha [6] and further investigated by Mielke [3,4] and Duran and Mielke [2]. It has been shown by these authors, under certain questionable assumptions, that this test has a higher asymptotic efficiency relative to the more conventional Wilcoxon test, that is, the sum of (first power) ranks test.

The effect of taking the squares of the ranks of the observations rather than the first powers is to attach much greater weight to the larger observations, and when there are hundreds of observations, as is not unusual in weather modification experiments, this means that the smaller observations are essentially ignored. Why this should be desirable is not intuitively clear.

It is the purpose of this paper to investigate the pertinence to reality of the underlying assumptions in the analysis of weather modification experiments by rank methods; and it will be shown under certain assumptions that for the usual objectives in weather modification and for the usual ranges of

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parameter values involved in such experiments, the sum of squared rank tests is not uniformly more efficient asymptotically than the Wilcoxon test.

All of the data used in this paper have been obtained from the Santa Barbara Convection Band Study. It is possible that some of the conclusions are not applicable to other areas, or other types of storms. We can only suggest that for each experiment in which rank tests are used the underlying assumptions be checked for validity from the data, as should be done in any case of course.

2. Assumptions. The fundamental assumption used in the theory given in justification of the use of the sum of squared ranks test in weather modification experiments is that the effect of cloud seeding is to produce a change in the scale of the rainfall distribution. The origin of this assumption has been attributed [2] to the single unsubstantiated statement by Scott and Neyman [5]:

> "...the opinion of certain knowledgable meteorologists may be quoted that, if the seeding of clouds has an effect on precipitation in the target, this effect is multiplicative. In other words, if for a given X=x the expected nonseeded precipitation in inches is $\eta(x)$ than the expected precipitation affected by seeding will be $\eta(x)e^{2\xi}$, where ξ stands for the positive or negative conventional measure of the effect of seeding."

Two questions arise here: firstly, is there evidence to support this belief, and secondly, if the belief is valid, what relevance does it have to the purpose of cloud seeding?

As far as data from the Santa Barbara Study indicates, there does not appear to be any particular reason to single out scale change as the only systematic distributional effect of cloud seeding. Each point in Figures 1 and 2 represents a randomly chosen rain gauge in Phase I of the Santa Barbara

Figure 1:

The scale parameter $\hat{\beta}_s$ of the gamma distribution fitted to the observed distribution of rainfall over all seeded bands at twenty-one randomly chosen locations in Phase I of the Santa Barbara Study plotted against the scale parameter $\hat{\beta}_u$ for unseeded bands at the same locations.

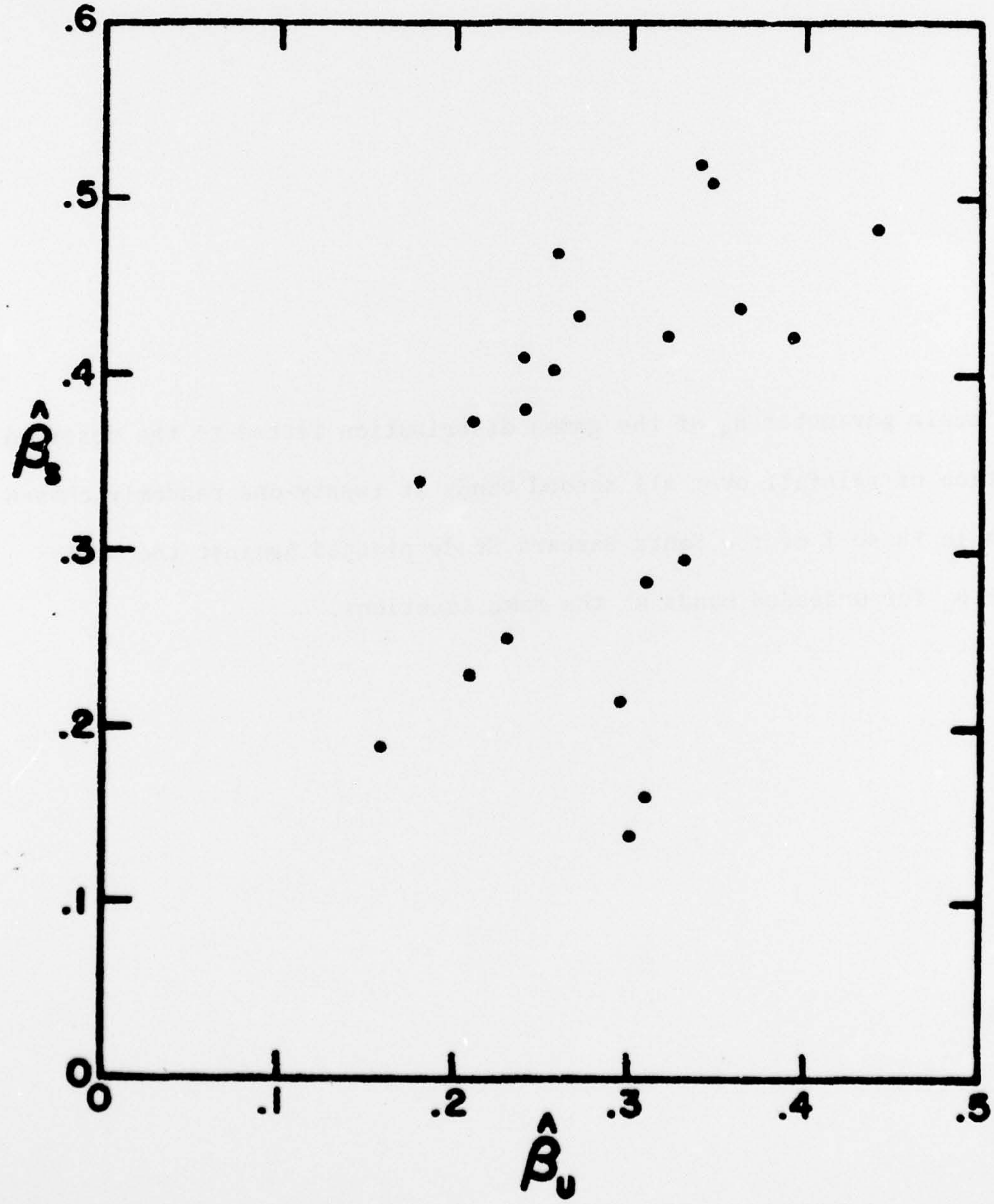
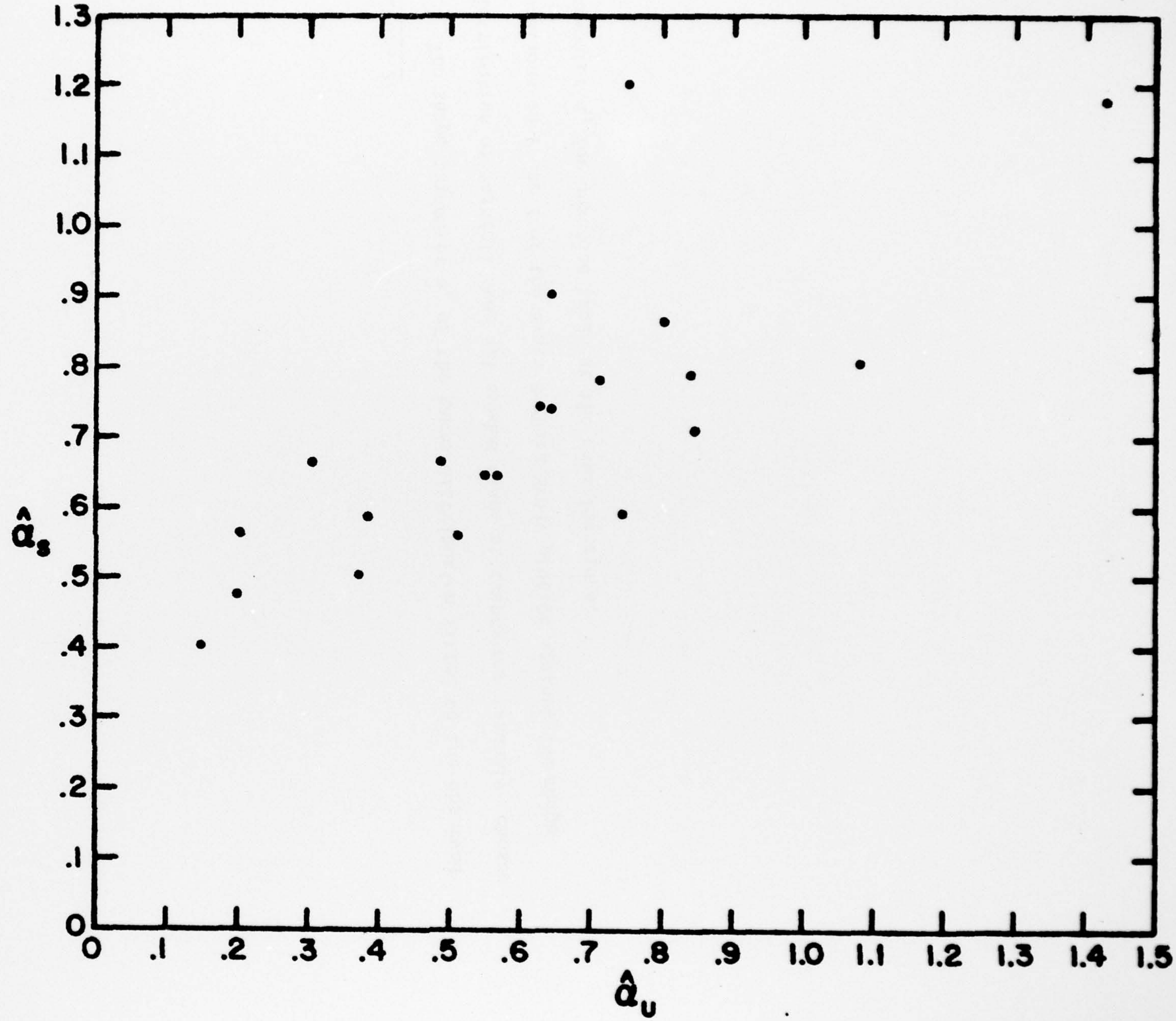


Figure 1.

Figure 2:

The shape parameter $\hat{\alpha}_s$ of the gamma distribution fitted to the observed distribution of rainfall over all seeded bands at twenty-one randomly chosen locations in Phase I of the Santa Barbara Study plotted against the shape parameter $\hat{\alpha}_u$ for unseeded bands at the same locations.



1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
$\hat{\alpha}_s$
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5
$\hat{\alpha}_u$

Figure 2.

Study. The ordinates $\hat{\beta}_u$ and $\hat{\beta}_s$ are the respective scale parameters, and the ordinates $\hat{\alpha}_u$ and $\hat{\alpha}_s$ are the respective shape parameters of gamma probability distribution functions ($f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}}$) fitted to the unseeded and seeded rainfall distributions, over all observed storms, at each experimental location. We see that the observed effect on the scale parameter is far less systematic and far more variable than it is on the shape parameter.

However, with regard to the second question raised above, the appropriate matter to consider is not the effect per se on either of these parameters. If we were interested in changes of scale, then perhaps it would not be unreasonable to use a test which emphasises the larger observations, where the absolute effect of scale change is greatest. A more usual matter of interest, though, would be the effect of seeding on average precipitation. (Note that the mean of the gamma distribution is $\alpha\beta$; so these two parameters are equally important as far as the mean is concerned.) Hence, if we desire to have a test based on a single parameter, it would be more appropriate to reparameterize the distribution, using the mean, say, as one of the parameters, and base our theory of test efficiency on the relevant parameter. So we write

$$f(x) = \frac{1}{\Gamma(\frac{\mu}{\beta})\beta^{\frac{\mu}{\beta}}} x^{\frac{\mu}{\beta}-1} e^{-\frac{x}{\beta}}$$

where μ is the mean of the gamma distribution, and β is retained as a free (nuisance) parameter since the effect of seeding on β is relatively much more variable than it is on α.

3. <u>Asymptotic Relative Efficiency</u>. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be independent random samples from two populations with cumulative distributions $F(x)$ and $G(x)$ respectively. The two-sample sum of r-th power rank

test statistic A_{Nr} is defined by

$$A_{Nr} = m^{-1} \sum_{i=1}^{N} \left(\frac{i}{N}\right)^r Z_{Ni}$$

where $N = m + n$, and $Z_{Ni} = 1$ or 0 if the i-th smallest value in the combined sample of X's and Y's is an X or a Y respectively.

We consider testing the null hypothesis

$$H_o: \ G(x) = F(x)$$

against an alternative of the form

$$H_1: \ G(x) = F(\phi(x,\theta))$$

for some specified cumulative distribution function F which has a density f defined on $[0,\infty)$; and for some specified function ϕ whose range is $[0,\infty)$ and which depends on a parameter θ such that $\phi(x, \theta_o) = x$, and that $\left[\frac{\partial\phi}{\partial\theta}\right]_{\theta=\theta_o}$ exists and is integrable on $[0,\infty)$. It is further assumed that the conditions of the Chernoff-Savage [1] asymptotic normality theorem for nonparametric test statistics hold.

Let $\mu_\theta(A_{Nr})$ and $\sigma_\theta^2(A_{Nr})$ denote the mean and variance of A_{Nr} under θ. Then the efficacy of the test statistic A_{Nr} is defined by

$$\varepsilon(A_{Nr}) = \frac{\left[\frac{\partial}{\partial\theta} \mu_\theta(A_{Nr}) \Big|_{\theta=\theta_o}\right]^2}{N\lambda_N(1-\lambda_N)\sigma_{\theta_o}^2(A_{Nr})}$$

where $\lambda_N = \frac{m}{N}$.

The asymptotic relative efficiency of the test statistic A_{Nr_2} with respect to the statistic A_{Nr_1} is defined by

$$e(A_{r_2}, A_{r_1}) = \lim_{N\to\infty} \frac{\varepsilon(A_{Nr_2})}{\varepsilon(A_{Nr_1})} .$$

Application of the asymptotic normality theorem for nonparametric test statistics given by Chernoff and Savage [1] gives

$$\mu_\theta\,(A_{Nr}) \sim \int_0^\infty [\lambda_N F(x) + (1 - \lambda_N)\, F(\phi(x,\theta))]^r dF(x)$$

and

$$\sigma_{\theta_0}^2\,(A_{Nr}) \sim \frac{r^2(1 - \lambda_N)}{(r + 1)^2(2r + 1)\, N\lambda_N}$$

where the sign $\sim$ indicates that the ratio of the two sides tends to unity as N tends to infinity.

Hence, we obtain

$$\left.\frac{\partial\mu_\theta(A_{Nr})}{\partial\theta}\right|_{\theta=\theta_0} \sim r(1 - \lambda_N) \int_0^\infty \left[\frac{\partial\phi}{\partial\theta}\right]_{\theta=\theta_0} f^2\, F^{r-1}\, dx$$

So the efficacy of A_{Nr} for large samples is

$$\varepsilon(A_{Nr}) \sim (r + 1)^2(2r + 1) \left[\int_0^\infty \left[\frac{\partial\phi}{\partial\theta}\right]_{\theta=\theta_0} f^2\, F^{r-1}\, dx\right]^2 .$$

Hence, for example, the asymptotic relative efficiency of the sum of squared ranks test with respect to the Wilcoxon test is given by

$$e(A_2, A_1) = \frac{15}{4} \left[\frac{\int_0^\infty \left[\frac{\partial\phi}{\partial\theta}\right]_{\theta=\theta_0} f^2\, F\, dx}{\int_0^\infty \left[\frac{\partial\phi}{\partial\theta}\right]_{\theta=\theta_0} f^2\, dx}\right]^2$$

These results do not assume any specific rainfall distribution.

4. Efficacy for the Gamma Distribution. If it is supposed that the unseeded and seeded rainfall distributions are gamma distributions with parameters (μ_o, β) and (μ, β) respectively, then the seeded rainfall y can be related to the unseeded rainfall x by the equation:

$$\frac{1}{\Gamma(\frac{\mu}{\beta})\,\beta^{\frac{\mu}{\beta}}} \int_o^y t^{\frac{\mu}{\beta}-1} e^{-\frac{t}{\beta}} dt = \frac{1}{\Gamma(\frac{\mu_o}{\beta})\,\beta^{\frac{\mu_o}{\beta}}} \int_o^x t^{\frac{\mu_o}{\beta}-1} e^{-\frac{t}{\beta}} dt.$$

In order to evaluate the expression for the efficacy obtained at the end of Section 3, we now seek an expression for the term $\left|\frac{\partial\phi}{\partial\theta}\right|_{\theta=\theta_o}$, that is, in terms of the notation used above, $\left|\frac{\partial y}{\partial\mu}\right|_{\mu=\mu_o}$. Differentiating the above equation with respect to μ, we obtain:

$$\frac{-1}{\left(\Gamma(\frac{\mu}{\beta})\,\beta^{\frac{\mu}{\beta}}\right)^2} \left[\frac{1}{\beta}\,\Gamma'(\tfrac{\mu}{\beta})\,\beta^{\frac{\mu}{\beta}} + \Gamma(\tfrac{\mu}{\beta})\,\frac{1}{\beta}\,\beta^{\frac{\mu}{\beta}}\ln\beta\right] \int_o^y t^{\frac{\mu}{\beta}-1} e^{-\frac{t}{\beta}} dt$$

$$+ \frac{1}{\Gamma(\frac{\mu}{\beta})\,\beta^{\frac{\mu}{\beta}}} \left[\frac{\partial y}{\partial\mu}\cdot y^{\frac{\mu}{\beta}-1} e^{-\frac{y}{\beta}} + \int_o^y \frac{1}{\beta}\, t^{\frac{\mu}{\beta}-1} \ln t\; e^{-\frac{t}{\beta}} dt\right] = 0.$$

So,

$$\frac{\partial y}{\partial\mu} = \frac{1}{\beta\, y^{\frac{\mu}{\beta}-1} e^{-\frac{y}{\beta}}} \int_o^y t^{\frac{\mu}{\beta}-1} e^{-\frac{t}{\beta}} \left[\frac{\Gamma'(\frac{\mu}{\beta})}{\Gamma(\frac{\mu}{\beta})} + \ln\beta - \ln t\right] dt\ .$$

Hence finally,

$$\left|\frac{\partial y}{\partial \mu}\right|_{\mu=\mu_o} = \frac{1}{\beta\, x^{\frac{\mu_o}{\beta}-1}\, e^{-\frac{x}{\beta}}} \int_o^x t^{\frac{\mu_o}{\beta}-1}\, e^{-\frac{t}{\beta}} \left| \frac{\Gamma'(\frac{\mu_o}{\beta})}{\Gamma(\frac{\mu_o}{\beta})} + \ln\beta - \ln t \right| dt \,.$$

Thus for the gamma distribution we have the efficacy of A_{Nr} for large samples as

$$\mathcal{E}(A_{Nr}) \sim \frac{(r+1)^2\,(2r+1)}{\beta\left(\Gamma(\frac{\mu_o}{\beta})\,\beta^{\frac{\mu_o}{\beta}}\right)^{r+1}} \left\{ \int_o^\infty \left(x^{\frac{\mu_o}{\beta}-1}\, e^{-\frac{x}{\beta}} \right.\right.$$

$$\times \int_o^x t^{\frac{\mu_o}{\beta}-1}\, e^{-\frac{t}{\beta}} \left| \frac{\Gamma'(\frac{\mu_o}{\beta})}{\Gamma(\frac{\mu_o}{\beta})} + \ln\beta - \ln t \right| dt$$

$$\left.\left. \times \left[\int_o^x t^{\frac{\mu_o}{\beta}-1}\, e^{-\frac{t}{\beta}}\, dt \right]^{r-1} \right) dx \right\}^2 .$$

5. Conclusion. The asymptotic efficiency $e(A_2, A_1)$ of the squared rank test relative to the Wilcoxon test for the range of values of $\frac{\mu_o}{\beta}$ occurring in Phase I of the Santa Barbara Study are given in Figure 3. We see that under the assumptions involved in the above theory, the squared rank test is more efficient for values of $\frac{\mu_o}{\beta}$ less than 0.72, but for greater values the Wilcoxon test is more efficient.

Figure 3:

The asymptotic efficiency [$e(A_2, A_1)$] of the squared rank test relative to the Wilcoxon test as a function of $\frac{\mu_o}{\beta}$, where μ_o is the mean and β is the scale parameter of the unseeded rainfall.

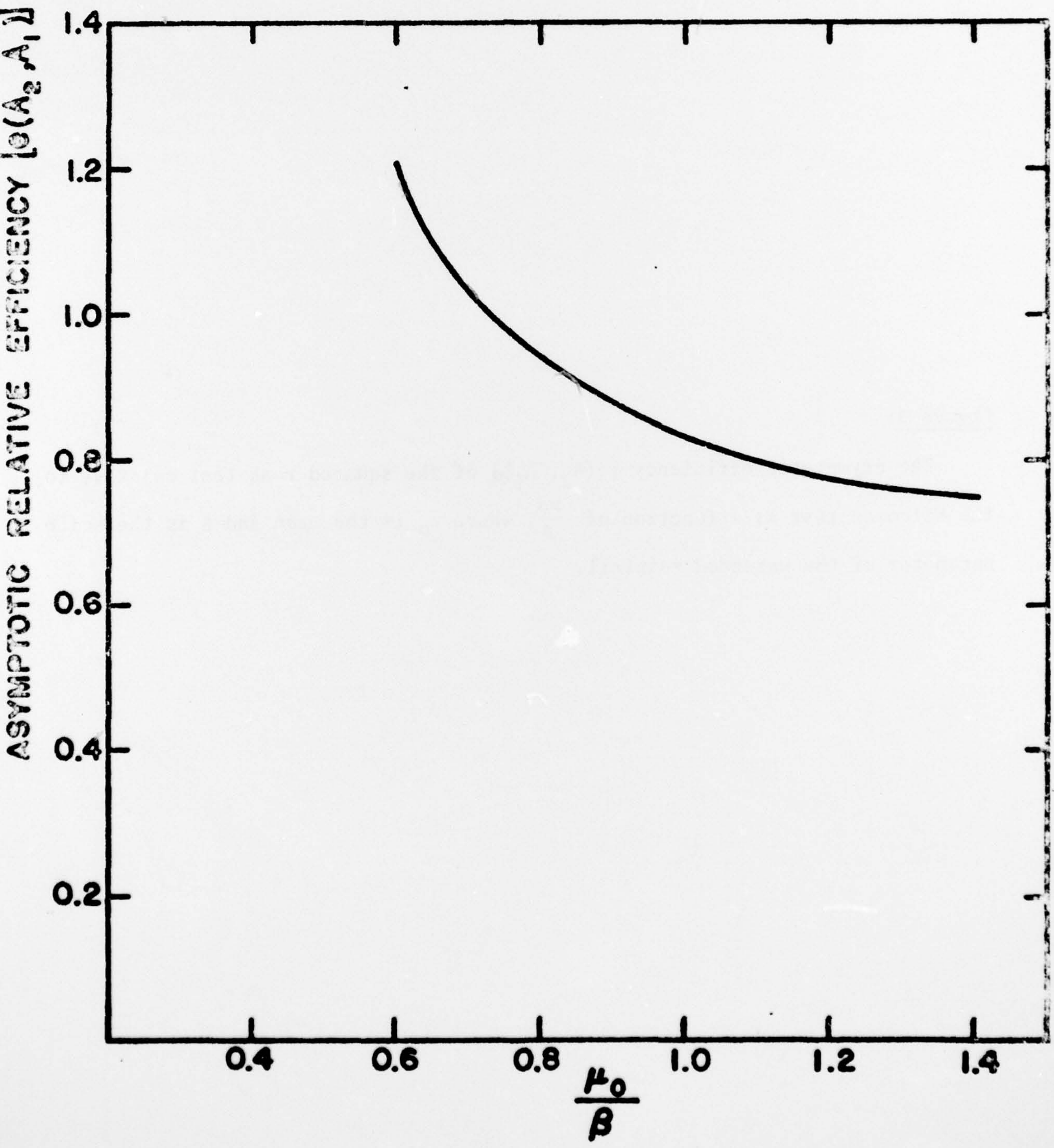

ASYMPTOTIC RELATIVE EFFICIENCY [e(A2,A1)]
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
μ0/β

Figure 3.

References

[1] H. Chernoff and I. R. Savage, "Asymptotic normality and efficiency of certain non-parametric test statistics," Ann. Math. Statist., 29 (1958), 972-994.

[2] B. S. Duran and P. W. Mielke, Jr., "Robustness of sum of squared ranks test," J. Amer. Statist. Assoc., 63 (1968), 338-344.

[3] P. W. Mielke, Jr., "Note on some squared rank tests with existing ties." Technometrics, 9 (1967) 312-314.

[4] P. W. Mielke, Jr., "Squared rank test appropriate to weather modification cross-over design," Technometrics, 16 (1974) 13-16.

[5] J. Neyman and E. L. Scott, "Asymptotically optimal tests of composite hypotheses for randomized experiments with noncontrolled predictor variables," J. Amer. Statist. Assoc., 60 (1965), 699-721.

[6] M. A. H. Taha, "Rank test for scale parameter for asymmetrical one-sided distributions. Publ. Inst. Statist. Univ. Paris, 13 (1964), 169-180.

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20. ABSTRACT

Rank tests are frequently used in the comparison of seeded and unseeded responses or of control and target responses in weather modification experiments. One of the more frequently used tests is the sum of squared ranks test, which under certain questionable assumptions, has been shown to have a higher asymptotic efficiency relative to the more conventional Wilcoxon test, that is the sum of (first power) ranks test. This paper investigates the pertinence to reality of the underlying assumption in the analysis of weather modification experiments by rank methods. It is shown that the squared rank test is not uniformly more efficient than the Wilcoxon test.